BFKL Pomeron at non-zero temperature and integrability of the Reggeon dynamics in multi-colour QCD

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Abstract

We consider the QCD scattering amplitudes at high energies $\sqrt{s}$ and fixed momentum transfers $\sqrt{-t}$ in the leading logarithmic approximation at a non-zero temperature T in the t-channel. It is shown that the BFKL Hamiltonian has the property of holomorphic separability. The Pomeron wave function for arbitrary T is calculated using an integral of motion. In multi-colour QCD, the holomorphic Hamiltonian for n-reggeized gluons at temperature T is shown to coincide with the local Hamiltonian of an integrable Heisenberg model and can be obtained from the $T=0$ Hamiltonian by an unitary transformation. We discuss the wave functions and the spectrum of intercepts for the colourless reggeon states.

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1. In QCD the scattering amplitudes $A(s,t)$ in Regge kinematics for high energies $2E = \sqrt{s}$ and fixed momentum transfers $q = \sqrt{-t}$ are obtained in the leading logarithmic approximation $\alpha_s \ln s \sim 1$, $\alpha_s = \frac{g^2}{4\pi} \to 0$ (g is the QCD coupling constant) by summing the largest contributions $\sim (\alpha_s \ln s)^n$ to all orders of perturbation theory within the approach of Balitsky, Fadin, Kuraev and Lipatov (BFKL) [1]. The BFKL Pomeron in the t-channel turns out to be a composite state of two reggeized gluons (it is valid also in the next-to-leading approximation [2]). Its wave function $\Psi(\vec{\rho}_1, \vec{\rho}_2)$ satisfies the Schrödinger equation in the two dimensional impact-parameter space $\vec{\rho}$

$$E\,\Psi(\vec{\rho}_1, \vec{\rho}_2) = H_{12}\,\Psi(\vec{\rho}_1, \vec{\rho}_2)\ . \tag{1}$$

The intercept Δ of the Pomeron, related to the high energy asymptotics $\sigma_t \sim s^{\Delta}$ of the total cross-section, is proportional to the ground state energy E

$$\Delta = -\frac{\alpha_s N_c}{2\pi}\,E\ .$$

The kinetic part $H_{kin} = \ln|p_1|^2 + \ln|p_2|^2$ of H_{12} is a sum of two gluon Regge trajectories and its potential part H_{pot} is related by a similarity transformation to the two-dimensional Green function $\ln|\rho_{12}|^2$, where $\rho_{12} = \rho_1 - \rho_2$. [We introduced here the complex coordinates $\rho_r = x_r + iy_r$ and the corresponding momenta $p_r = i\partial_r$].

The BFKL equation is used for the description of the deep-inelastic lepton-hadron scattering together with the DGLAP equation [3] (see for example [4]). It is invariant under the Möbius transformations [5]

$$\rho_r \to \frac{a\rho_r + b\rho_r}{c\rho_r + d\rho_r}$$

with arbitrary complex parameters a, b, c, d and H_{12} has the property of holomorphic separability (see [4] [6])

$$H_{12} = h_{12} + h_{12}^*\,,\ \ h_{12} = \sum_{r=1}^{2}\left[\ln\,p_r + \frac{1}{p_r}\,\ln\left(\rho_{12}\right)\,p_r - \psi(1)\right]\ , \tag{2}$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$.

The wave functions Ψ belong to the principal series of unitary representations of the Möbius group with conformal weights $m = 1/2 + i\nu + n/2$, $\widetilde{m} = 1/2 + i\nu - n/2$ expressed in terms of the anomalous dimension $\gamma = 1 + 2i\nu$ and the integer conformal spin n for the local gauge-invariant operators [5]. The conformal weights are related to the eigenvalues $m(m-1)$, $\widetilde{m}(\widetilde{m}-1)$ of the Casimir operators M^2 and M^{2*}, where

$$M^2 = \left(\sum_{r=1}^{2} M_3^{(r)}\right)^2 + \frac{1}{2}\left(\sum_{r=1}^{2} M_+^{(r)}\,\sum_{s=1}^{2} M_-^{(s)} + \sum_{r=1}^{2} M_-^{(r)}\,\sum_{s=1}^{2} M_+^{(s)}\right) = \rho_{12}^2\,p_1\,p_2\ .$$

Here $\vec{M}^{(r)}$ are the Möbius group generators

$$M_3^{(r)} = \rho_r\,\partial_r\,,\ \ M_+^{(r)} = \partial_r\,,\ \ M_-^{(r)} = -\rho_r^2\,\partial_r$$

and $\partial_r = \partial/\partial\rho_r$.

The eigenfunctions of H_{12} can be considered as the three-point functions of a two-dimensional conformal field theory and have the property of holomorphic factorization [5],

$$f_{m,\widetilde{m}}(\overrightarrow{\rho_1},\overrightarrow{\rho_2};\overrightarrow{\rho_0}) = \langle 0|\ \varphi(\overrightarrow{\rho_1})\,\varphi(\overrightarrow{\rho_1})\,O_{m,\widetilde{m}}(\overrightarrow{\rho_0})\ |0\rangle = \left(\frac{\rho_{12}}{\rho_{10}\,\rho_{20}}\right)^m \left(\frac{\rho_{12}^*}{\rho_{10}^*\,\rho_{20}^*}\right)^{\widetilde{m}} . \tag{3}$$

One can calculate the energy putting this Ansatz in the BFKL equation[1]

$$E_{m,\widetilde{m}} = \varepsilon_m + \varepsilon_{\widetilde{m}} \quad , \quad \varepsilon_m = \psi(m) + \psi(1-m) - 2\psi(1)\ . \tag{4}$$

The minimum of $E_{m,\widetilde{m}}$ is obtained at $m = \widetilde{m} = 1/2$ leading to a large intercept $\Delta = 4\ \frac{\alpha_s}{\pi} N_c \ln 2$ of the BFKL Pomeron. In the next-to-leading approximation the intercept is comparatively small ($\Delta \sim 0.2$ for the QCD case) [7].

2. On the other hand, now a significant interest is devoted to the quark-gluon plasma (QGP) generation in heavy nucleus collisions (see for example [9]). Current theoretical understanding suggests that the QGP thermalizes via parton-parton scattering. The QGP is understood to cool down by hydrodynamic expansion till the temperature reaches the hadronization scale $\sim$ 160MeV. One interesting phenomena is the suppression of the ψ-meson production in the heavy nucleus collisions due to the disappearance of the confining potential between q and $\bar{q}$ at high temperature [9]. A similar effect should exist for glueballs constructed from gluons. Because the Pomeron is considered as a composite state of reggeized gluons, the influence of the temperature on its properties is of great interest. In this paper we construct the BFKL equation at temperature T in the center-mass system of the t -channel (where $\sqrt{t} = 2\epsilon$) and investigate the integrability properties of the BFKL dynamics in a thermostat for composite states of n reggeized gluons in multi-colour QCD.

Let us consider the Regge kinematics in which the total particle energy $\sqrt{s}$ is asymptotically large in comparison with the temperature T. In this case one can neglect the temperature effects in the propagators of the initial and intermediate particles in the direct channels s and u. But the momentum transfer $|q|$ is considered to be of the order of T (note, that q_μ is the vector orthogonal to the initial momenta $q_\mu \approx q_\mu^\perp$). As it is well known [8], the particle wave functions $\psi(x_\mu)$ at temperature T are periodic in the euclidean time $\tau = i\,t$ with period $1/T$.

We introduce the temperature T in the center of mass frame of the t-channel. Thus, the euclidean energies of the intermediate gluons in the t-channel become quantized as

$$k_4^{(l)} = 2\pi l\,T\ .$$

In the s-channel the invariant t is negative and therefore the analytically continued 4-momenta of the t-channel particles can be considered as euclidean vectors. It means, that at temperature T, the wave functions for virtual gluons are periodic functions of the holomorphic impact-parameter $\rho = x+iy$ with imaginary period $\frac{i}{T}$. Also, the canonically conjugated momenta p have their imaginary part quantized,

$$\rho \to \rho + \frac{i}{T} \quad , \quad p = \mathrm{Re}\,p + \pi\,i\,l\,T\ . \tag{5}$$

with integer l (note that $p = (p_1 + ip_2)/2$).

It is convenient to rescale the transverse coordinates and corresponding momenta as follows

$$\rho \to \frac{1}{2\pi\,T}\,\rho \quad , \quad p \to 2\pi\,T\,p\ .$$

In these dimensionless variables one obtains

$$0 < \mathrm{Im}\,\rho < 2\pi \quad , \quad \mathrm{Im}\,p = \frac{l}{2}\,.$$

The calculation of the Regge trajectory $1+\omega(t)$ of the gluon at temperature T in the t-channel, in one-loop approximation reduces to the integration over the real part k_1 of the transverse momentum $\vec{k}_\perp$ of the virtual gluon and to the summation over its imaginary part $k_2 = l$. In such a way we obtain the following result for the trajectory having the separability property [cf. [6]],

$$\omega(-\vec{q}^{\,2}) = -\frac{g^2}{8\pi^2}\, N_c\, \Omega(-\vec{q}^{\,2}) \quad , \quad \Omega(-\vec{q}^{\,2}) = \Omega(q) + \Omega(q^*)\,.$$

Here,

$$\Omega(q) = \frac{\pi T}{\lambda} + \frac{1}{2}\,\left[\psi(1+iq) + \psi(1-iq) - 2\psi(1)\right]\,,$$

where we regularized the infrared divergence for the zero mode $l = 0$ introducing a mass λ for the gluon (see [1]).

A similar divergence appears in the Fourier transformation $G(\overrightarrow{\rho}_{12})$ of the effective gluon propagator $(\vec{k}_\perp^2 + \lambda^2)^{-1}$ contained in the product of the effective vertices $q_1 k^{-1} q_2^*$ for the production of a gluon with momentum k_μ (cf. [4])

$$G(\overrightarrow{\rho}_{12}) = -\frac{\pi\, T}{\lambda} + \ln\left(2\sinh\frac{\rho_{12}}{2}\right) + \ln\left(2\sinh\frac{\rho_{12}^*}{2}\right)\,.$$

Therefore, the divergence at $\lambda \to 0$ cancels in the sum of kinetic and potential contributions to the BFKL equation and the Hamiltonian H_{12} for the Pomeron in a thermostat has the property of holomorphic separability with the holomorphic Hamiltonian given below [cf. eq.(2)]

$$h_{12} = \sum_{r=1}^{2}\left[\frac{1}{2}\,\psi(1+ip_r) + \frac{1}{2}\,\psi(1-ip_r) + \frac{1}{p_r}\,\ln\left(2\sinh\frac{\rho_{12}}{2}\right)\,p_r - \psi(1)\right]\,. \qquad (6)$$

3. The Hamiltonian (6) is a periodic function of ρ_{12}. Therefore its eigenfunctions are quasi-periodic functions of this variable. The behaviour of h_{12} for small ρ_{12} corresponds to the low temperature regime. Hence, the eigenfunctions of h_{12} behave for $\rho_{12} \to 0$ as the holomorphic part of the zero-temperature wave functions eq.(3),

$$\Psi_m(\rho_{12}) \overset{\rho_{12}\to 0}{=} \rho_{12}^m \qquad (7)$$

Notice that $\Psi_{1-m}(\rho_{12})$ is an eigenfunction too.

We find the small-T expansion of h_{12} near its singularities $\rho_{12} = 2\pi i l$. For example, for small ρ_{12} and large p_1, p_2 we have

$$h_{12} = h_{12}^0 + \sum_{k=1}^{\infty}\frac{B_{2k}}{2k}\sum_{r=1,2}\left[\frac{(-1)^{k+1}}{p_r^{2k}} + \frac{1}{p_r}\,\frac{\rho_{12}^{2k}}{(2k)!}\,p_r\right]\,,$$

where h_{12}^0 is the holomorphic BFKL Hamiltonian at a zero temperature [given by eq.(2)] and the B_{2k} are the Bernoulli numbers. This representation for h_{12} permits us to find the

small-T expansion of its eigenfunctions. For example, at a vanishing momentum transfer $Q = p_1 + p_2$ we obtain for the eigenfunction $\Psi_m(\rho_{12})$,

$$\Psi_m(\rho_{12}) = \rho_{12}^m \left[1 - \frac{1}{24}\frac{m(m-1)}{2m+1}\,\rho_{12}^2 + \frac{1}{5760}\frac{m(m-1)(5m^2+7m+6)}{(2m+1)(2m+3)}\,\rho_{12}^4 + \mathcal{O}(\rho_{12}^6)\right].$$

These eigenfunctions are parametrized by the conformal weight m.

On the other hand, from the above expansion it is possible to verify, that the holomorphic Hamiltonian has the non-trivial integral of motion:

$$A = 4\sinh^2\frac{\rho_{12}}{2}\,p_1\,p_2 \quad , \quad [A, h_{12}] = 0\,. \tag{8}$$

Therefore, instead of solving the Schrödinger equation we can search for the eigenfunctions of the operator A. For non-zero Q one can write the holomorphic wave function as a product of a plane wave depending on $R = (\rho_1 + \rho_2)/2$ times a solution of the following equation for the relative motion of two gluons

$$\left[\frac{Q^2}{4} + \frac{\partial^2}{\partial\rho^2}\right]\,\Psi(\rho, Q) = \frac{m(m-1)}{4\sinh^2\frac{\rho}{2}}\,\Psi(\rho, Q) \quad , \quad \rho = \rho_{12}\,,\ t = -4\,|Q|^2\ .$$

The two independent solutions of the above differential equation can be expressed in terms of hypergeometric functions

$$\Psi_1^{(m)}(\rho,\,Q) = e^{\frac{i}{2}Q\,\rho}\,\left(e^\rho - 1\right)^m\,F\left(iQ + m, m; 2m; 1 - e^\rho\right) \quad , \quad \Psi_2^{(m)}(\rho,\,Q) \equiv \Psi_1^{(1-m)}(\rho,\,Q)\,. \tag{9}$$

For $\rho \to 0$ eq.(7) holds and the singularities of $\Psi^{(r)}(\rho,\,Q)$ at $1 - e^\rho = 1$ and $1 - e^\rho = \infty$ correspond to the points $\rho = -\infty$ and $\rho = \infty$, respectively.

The analytic continuation of $\Psi^{(r)}$ along the imaginary axes from $\rho = 0$ to $\rho = 2\pi i$ is equivalent to the continuation of these eigenfunctions in a circle passed in a clock-wise direction around the singularity at $\rho = -\infty$. The monodromy matrix expressing the analytically continued solutions in terms of the initial ones can be easily calculated.

The Pomeron wave functions can be written as a bilinear combination of holomorphic and anti-holomorphic eigenfunctions $\Psi^{(r)}(\rho,\,Q)$ and $\Psi^{(r)}(\rho^*,\,Q^*)$. The property of single-valuedness in the cylinder topology corresponding to the periodicity on the boundaries of the strip $0 < \operatorname{Im}\rho_{12} < 2\pi$ is easily imposed to such Pomeron wave function using the monodromy matrix for $\Psi^{(r)}(\rho,\,Q)$. The resulting wave function can be written as,

$$\Psi^{(m,\widetilde{m})}(\vec{\rho},\,\vec{Q}) = \chi_1^{(m)}(\rho,\,Q)\,\chi_1^{(\widetilde{m})}(\rho^*,\,Q^*) - (-1)^N\,\chi_2^{(m)}(\rho,\,Q)\,\chi_2^{(\widetilde{m})}(\rho^*,\,Q^*)\,, \tag{10}$$

where,

$$\chi_1^{(m)}(\rho,\,Q) = 2^{1-2\,m}\,\frac{\Gamma\left(m + iQ\right)}{\Gamma\left(m + \frac{1}{2}\right)}\,\Psi_1^{(m)}(\rho,\,Q)\,,\ \chi_2^{(m)}(\rho,\,Q) = \chi_1^{(1-m)}(\rho,\,Q)$$

and $N = 2\operatorname{Im}Q$ is an integer.

4. The Pomeron wave function can be constructed directly in coordinate space. For this purpose we use the conformal transformation

$$\rho_r = \ln\,\rho_r' \tag{11}$$

and the integral of motion eq.(8) becomes

$$A = -(\rho'_{12})^2 \frac{\partial}{\partial\rho'_1}\frac{\partial}{\partial\rho'_2}\,.$$

Thus, A coincides in the variables ρ'_r with the Casimir operator of the conformal group whose eigenfunctions are well known (see [5]). Thus, the Pomeron wave function at non-zero temperature having the property of single-valuedness and periodicity takes the form

$$\Psi^{(m,\widetilde{m})}(\vec{\rho_1},\,\vec{\rho_2},\,\vec{\rho_0}) = \left(\frac{\sinh\frac{\rho_{12}}{2}}{2\sinh\frac{\rho_{10}}{2}\,\sinh\frac{\rho_{20}}{2}}\right)^m \left(\frac{\sinh\frac{\rho^*_{12}}{2}}{2\sinh\frac{\rho^*_{10}}{2}\,\sinh\frac{\rho^*_{20}}{2}}\right)^{\widetilde{m}}\,. \qquad (12)$$

The orthogonality and completeness relations for these functions can be easily obtained from the analogous results for $T = 0$ (see [5]) using the above conformal transformation. These wave functions are proportional to the Fourier transformation of the wave functions $\Psi^{m,\widetilde{m}}(\vec{\rho},\vec{Q})$.

Moreover, the pair BFKL Hamiltonian h_{12} can be expressed in terms of the BFKL Hamiltonian at zero temperature in the new variables

$$h_{12} = \ln(p'_1\,p'_2) + \frac{1}{p'_1}\,\log(\rho'_{12})\,p'_1 + \frac{1}{p'_2}\,\log(\rho'_{12})\,p'_2 - 2\psi(1)\,, \qquad (13)$$

where $p'_r = i\frac{\partial}{\partial\rho'_r}$. In the course of the derivation the following operator identity (see [4])

$$\frac{1}{2}\left[\psi\left(1+z\frac{\partial}{\partial z}\right)+\psi\left(-z\frac{\partial}{\partial z}\right)\right] = \ln\,z + \ln\,\frac{\partial}{\partial z}$$

was used to transform the kinetic part as well as properties of the ψ-function.

In summary, the exponential mapping eq.(11) which in dimensional variables takes the form

$$\rho' = \frac{1}{2\pi\,T}\,e^{2\pi\,T\,\rho}\,,$$

maps the reggeon dynamics from zero temperature to temperature T. This mapping explicitly exhibits a periodicity $\rho \to \rho + \frac{i}{T}$ for a thermal state. It must be noticed that such class of mappings are known to describe thermal situations for quantum fields in accelerated frames and in black hole backgrounds[23].

5. As it is well known [10], the BFKL equation at $T = 0$ can be generalized to composite states of n reggeized gluons. In the multi-colour limit $N_c \to \infty$ the BKP equations are significantly simplified thanks to their conformal invariance [5], holomorphic separability [6] and integrals of motion [11]. The generating function for the holomorphic integrals of motion coincides with the transfer matrix for an integrable lattice spin model [12] [13]. The transfer matrix is the trace of the monodromy matrix

$$t(u) = L_1(u)\,L_2(u)...L_n(u)\,,$$

satisfying the Yang-Baxter equations [13]. The integrability of the n-reggeon dynamics in multi-colour QCD is valid also at non-zero temperature T, where, according to the above arguments we should take the L-operator in the form

$$L_k = \begin{pmatrix} u+p_k & e^{-\rho_k}\,p_k \\ -e^{\rho_k}\,p_k & u-p_k \end{pmatrix}.$$

In particular, the holomorphic Hamiltonian is the local Hamiltonian of the integrable Heisenberg model with the spins being unitarily transformed generators of the Möbius group (cf. [14] [15])

$$M_k = \partial_k \quad , \quad M_+ = e^{-\rho_k}\, \partial_k \quad , \quad M_- = -e^{\rho_k}\, \partial_k \; .$$

Because the Hamiltonian at non-zero temperature can be obtained by an unitary transformation from the zero temperature Hamiltonian, the spectrum of the intercepts for multi-gluon states is the same as for zero temperature [16]- [20] and the wave functions of the composite states can be calculated by the substitution $\rho_k \to e^{\rho_k}$.

Furthermore, the non-linear Balitsky-Kovchegov equation [21] can be generalized to the case of non-zero temperature as follows,

$$\frac{\partial N_{\vec{\rho}_1,\vec{\rho}_2}}{\partial Y} = \bar{\alpha}_s \int \frac{d^2\rho_0}{2\pi}\, \frac{\left|\sinh \frac{\rho_{12}}{2}\right|^2}{4\left|\sinh \frac{\rho_{10}}{2}\right|^2 \left|\sinh \frac{\rho_{20}}{2}\right|^2}\, \left(N_{\vec{\rho}_1,\vec{\rho}_0} + N_{\vec{\rho}_2,\vec{\rho}_0} - N_{\vec{\rho}_1,\vec{\rho}_2} - N_{\vec{\rho}_1,\vec{\rho}_0}\, N_{\vec{\rho}_2,\vec{\rho}_0}\right) , \quad (14)$$

where $N_{\vec{\rho}_1,\vec{\rho}_2}$ is the amplitude of finding a dipole with the impact parameters $\vec{\rho}_1$ and $\vec{\rho}_2$ in a hadron and the integration over ρ_0 is performed over the strip $0 < \mathrm{Im}\,\rho_0 < 2\pi$. Note, however, that in this equation one takes into account only fan diagrams for the Pomeron interactions among all possible diagrams for reggeized gluons appearing in the high energy effective action [22].

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References

[1] L. N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 642;
V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. **B60** (1975) 50; Sov. Phys. JETP **44** (1976) 443; **45** (1977) 199;
Ya. Ya. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.

[2] V. S. Fadin, L.N. Lipatov, Phys. Lett. **B429** (1998) 127;
G. Camici and M. Ciafaloni, Phys. Lett. **B430** (1998) 349.;
A. V. Kotikov, L. N. Lipatov, Nucl. Phys. **B582** (2000); hep-ph/0208220.

[3] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **18** (1972) 438, 675;
L. N. Lipatov, Sov. J. Nucl. Phys. **20** (1975) 93;
G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298;
Yu. L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641.

[4] L. N. Lipatov, *Pomeron in QCD*, in 'Perturbative QCD', ed. by A. N. Mueller, World Scientific, 1989; *'Small-x physics in the perturbative QCD'*, Phys. Rep., **286** (1997) 132.

[5] L. N. Lipatov, Sov. Phys. JETP **63** (1986) 904.

[6] L. N. Lipatov, Phys. Lett. **B251** (1990) 284.

[7] S. J. Brodsky, V. S. Fadin, V. T. Kim, L. N. Lipatov, G. V. Pivovarov, JETP Letters **B70** (1999) 155; **B76** (2002) 306.

[8] See for example, J. I. Kapusta, *Finite Temperature Field Theory*, Cambridge Univ. Press, 1989; M. Le Bellac, *Thermal Field Theory*, Cambridge University Press, 1996.

[9] *Quark-Gluon Plasma 3*, edited by R. C. Hwa and X. N. Wang, World Scientific, September 2003. E. V. Shuryak, 'The QCD vacuum, hadrons and superdense matter', World Sci. Lect. Notes Phys. 8: 1-401, 1988. H. Satz, Rept. Prog. Phys. 63, 1511 (2000). See also, E. V. Shuryak, I. Zahed, hep-ph/0307267.

[10] J. Bartels, Nucl. Phys. **B175** (1980) 365;
J. Kwiecinski and M. Prascalowicz, Phys. Lett. **B94** (1980) 413.

[11] L. N. Lipatov, Phys. Lett. **B309** (1993) 394.

[12] L. N. Lipatov, Proceedings of the Fifth Blois Workshop, edited by H.M. Fried, K.Kang, C-I Tan, June 1993, World Scientific.

[13] L. N. Lipatov, hep-th/9311037, Padova preprint DFPD/93/TH/70, unpublished.

[14] L. N. Lipatov, Sov. Phys. JETP Lett. **59** (1994) 571.

[15] L. D. Faddeev, G. P. Korchemsky, Phys. Lett. **342** (1995) 311.

[16] R. Janik and J. Wosiek, Phys. Rev. Lett. **79** (1997) 2935; **82** (1999) 1092.

[17] L. N. Lipatov, Nucl. Phys. **B548** (1999) 328.

[18] J. Bartels, L. N. Lipatov, G. P. Vacca, Phys.Lett. **B477** , 178 (2000).

[19] H. J. de Vega and L. N. Lipatov, Phys. Rev. **D64**, 114019 (2001); **D66**, 074013 (2002).

[20] A. Derkachev, G. Korchemsky, A. Manashov, Nucl. Phys. **B617** (2001) 375.

[21] I. Balitsky, Phys. Rev. **D60,** 014020 (1999);
Y. V. Kovchegov, Phys. Rev. **D61,** 074018 (2000).

[22] L. N. Lipatov, Nucl. Phys. **B452,** (1995) 369.

[23] See for example, N. D. Birrell, P. C. W. Davis, 'Quantum Fields in Curved Space', Cambridge U. Press, 1982 and references therein. S. W. Hawking, Comm. Math. Phys. 43, 199 (1975). N. Sanchez, Phys. Lett. B87, 212 (1979), Phys. Rev. D24, 2100 (1981).